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Answer Catch up exam Maths 2 *lasts 1:30 minutes* *02/06/2024*

Name:...............
Group:..............

Exercise 1 ***(9 pts) Part I*** *Choose the right answer. Let*

$$I = \int_1^e \frac{1}{x}\ln\frac{1}{x}dx\ ,\quad J = \int \frac{1}{x^2+2x-3}dx\ ,\ K = \int x\sin x dx\ ,$$
$$L = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}dx \quad and \quad M = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x}dx$$

1- *The value of I is*

☒ $I = -\frac{1}{2}$. $I = \frac{1}{2}$. $I = e$. $I = 1$

2- *The integral J is*

. $J = \ln\left|x^2+2x+3\right| + C$. $J = \ln(x+1) - \ln(x+3) + C$ ☒ $J = \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

3- *The integral K is*

☒ $\mathbf{K} = \sin x - x\cos x + C$. $\mathbf{K} = \frac{1}{2}x^2\cos x + C$. $K = \sin x + x\cos x + C$

4. *The value of* $L+M$ *and* $L-M$ *are*

. $\begin{cases} L+M=0 \\ L-M=\frac{\pi}{2}\end{cases}$ ☒ $\begin{cases} L+M=\frac{\pi}{2} \\ L-M=0\end{cases}$. $\begin{cases} L+M=\frac{\pi}{2} \\ L-M=\frac{\pi}{2}\end{cases}$. $\begin{cases} L+M=\frac{\pi}{2} \\ L-M=-\frac{\pi}{2}\end{cases}$

5. *The value of K and L are*

☒ $\begin{cases} L=\frac{\pi}{4} \\ M=\frac{\pi}{4}\end{cases}$. $\begin{cases} L=-\frac{\pi}{4} \\ M=\frac{\pi}{4}\end{cases}$. $\begin{cases} L=0 \\ M=\frac{\pi}{2}\end{cases}$. $\begin{cases} L=\frac{\pi}{2} \\ M=-\frac{\pi}{2}\end{cases}$

Part II *Let the following diferentiabl equations*

$$y' + xy = x^2+1 \quad (1)$$
$$2y'' - 3y' + y = 0 \quad (2)$$

6. *The homogenous somutions of 1 are*

. $y_H = Ce^{-x}$. $y_H = Ce^{x}$ ☒ $y_H = Ce^{-\frac{1}{2}x^2}$. $y_H = Ce^{\frac{1}{2}x^2}$

7- *The partucular somution of 1 is*

. $y_p = 1$ ☒ $y_p = x$. $y_p = xe^x$. $y_p = -x$

8- *The homogenous solutions of 2 are*

☒ $y_H = C_1e^x + C_2e^{\frac{1}{2}x}$. $y_H = C_1e^x + C_2e^{-x}$. $y_H = C_1e^x + C_2e^{-\frac{1}{2}x}$

9. *The limited developement of* $f(x) = (e^{-x} + x)(x + \ln(1 + x))$ *in order* 2 *is*

☒ $f(x) = 2x - \frac{1}{2}x^2 + x^2\varepsilon(x)$. $f(x) = 2x + \frac{1}{2}x^2 + x^2\varepsilon(x)$. $f(x) = x - \frac{1}{2}x^2 + x^2\varepsilon(x)$

Exercise 2 ***6 pts*** *Consider the following map* $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ *and the matrix* P *defined by*

$$f(x, y) = (3x - 2y, x) \text{ and } P = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}.$$

1. *The matrix* $A = M_B(f)$ *representing* f *with respect to the standard bases* B *of* $\mathbb{R}^2$ *is*

☒ $A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$. $A = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$. $A = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

2- *The inverse matrix of* P *is*

☒ $P^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$. $P^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$. $P^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$

3- *The result of* $A^2 - 3A$ *is*

. $A^2 - 3A = 2I_2$. $A^2 - 3A = I_2$ ☒ $A^2 - 3A = -2I_2$

4. *The eigenvalues of* A *are*

. $\lambda_1 = 1$ *or* $\lambda_2 = -2$ ☒ $\lambda_1 = 1$ *or* $\lambda_2 = 2$. $\lambda_1 = -1$ *or* $\lambda_2 = 2$

5. *The result of* $P^{-1}AP$ *is*

☒ $P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. $P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$. $P^{-1}AP = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

6. *For all* $n \geq 1$, *we have*

. $A^n = \begin{pmatrix} 2 \times 2^n - 1 & 2 - 2 \times 2^n \\ 2^n - 1 & 2 - 2^n \end{pmatrix}$ ☒ $A^n = \begin{pmatrix} 2 \times 2^n + 1 & 2 - 2 \times 2^n \\ 2^n - 1 & 2 - 2^n \end{pmatrix}$. $A^n = \begin{pmatrix} 3^n & -2^n \\ 1 & 0 \end{pmatrix}$

Exercise 3 ***5 pts*** *Determine the solution of the following differential equation*

$$\begin{cases} xy' + y = xy^3 \\ y(1) = \sqrt{3} \end{cases} \tag{3}$$

Solution 4 *This is a Bernoulli differentiable equation* **??** *, where $\alpha = 3$. We first divided the equation throught by y^3, thereby expressing it in the equivalent form*

$$x\frac{y'}{y^3} + \frac{1}{y^2} = x \text{.......\textbf{\textit{1pts}}} \tag{4}$$

by using the change variable $z = y^{1-\frac{1}{2}}$, then $z' = \frac{1}{2}\frac{y'}{\sqrt{y}}$ and equation 4 transforms into

$$xz' - 2z = -2x \text{.......\textbf{\textit{1pts}}} \tag{5}$$

the solution of linear differential equation of 1st order 5 is

$$z = Cx^2 + 2x \text{.......\textbf{\textit{1pts}}}$$

Thus we obtain the solutions of **??** *in then form*

$$y = \frac{1}{\sqrt{Cx^2 + 2x}} \text{.......\textbf{\textit{1pts}}}$$

since $y(1) = \sqrt{3}$, we have

$$\frac{1}{\sqrt{C+2}} = \sqrt{3} \Longleftrightarrow C = -\frac{5}{3}$$

finaly the solution of is

$$y = \frac{1}{\sqrt{\frac{-5}{3}x^2 + 2x}} \text{.......\textbf{\textit{1pts}}}$$